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Generation principle for the robust comb-like resonant modes in photonic crystal ring resonator

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Topological valley photonic crystal cavity supports the robust comb-like resonant modes, and thus is widely applied in chiral coupling of quantum dots, laser, nonlinear frequency conversion process, and photonic logical elements. So far, its robustness is mostly attributed to the different topological properties of the valley photonic crystals. In this work, we illustrate that the position of circular polarization topological singularity is the most important factor to generate comb-like resonant modes with robustness. We proposed a photonic crystal ring resonator purely by the line-defect waveguide, and simulation results demonstrate that the proposed cavity can generate robust comb-like resonant modes. This work can provide guidance to the design of high-quality and robust photonic crystal cavity.

I. INTRODUCTION

Topological photonics is an important branch of modern photonics. Topological photonic devices grant electromagnetic waves the ability to propagate regardless of disorder. Since topological photonic waveguide is realized in gyromagnetic photonic crystal (PhC) [1, 2], numerous topological photonic structures are proposed like PhC based on the photonic quantum spin-Hall effect [3]. Topological valley photonic crystal (VPC) waveguide has become a hot spot recently owing to the unique superiority of passing Z-shape sharp bends over a large bandwidth [4, 5]. Compared with PhC based on the photonic quantum spin-Hall effect, the edge state in VPC exhibits no out-of-plane loss. For the physical mechanism of the formation of the edge state, related theories are constantly updated. The widely used explanation is the different topological properties of the VPCs in different parts. The topological property can be characterized by the valley Chern number (VCN) [5], and the opposite VCNs ensure that the intervalley scattering is suppressed which brings robust transmission. However, after then the broadband efficient transmission of the sharp bend is discovered in the interface constructed by PhCs with the zero VCNs [6, 7], which indicates that the suppression of the intervalley scattering and robust transmission against sharp bend are not closely connected. These researches illustrated that the waveguide with the broken inversion symmetry and zero VCNs may generate the edge state as well. Followed by these studies, robust transmissions in the line-defect photonic crystal are discovered [8, 9]. These devices exhibit no photonic valley-Hall effect, and lack the inversion symmetry [8, 9]. Moreover, the suppression of the backscattering against the disorder of the rods is found to be invalid in slow-light VPC waveguide [10, 11]. Researchers believe the efficient transmission of the sharp bend in VPC is a just-so configuration [12].

Recently, researchers illustrated two physical mecha-

nisms to explain the robust transmission of VPC. One is based on the Fourier transform of the field in the projected band, and the other is based on the Stokes parameter distribution [13, 14]. The former attributes the robust transmission to the overlapping of the K point in the Brillouin zone and the intense field in the momentum space transformed from the projected band eigen mode profile [14]. The latter attributes the robust transmission to the crossing of the circular polarization singularity (CPS), the intense field of the projected band eigen mode profile, and the symmetry axis of the PhC waveguide [13]. The CPS locates at the intersection of $S_1=0$ contour line and $S_2=0$ contour line, where S_1 and S_2 are Stokes parameters [13]. Researchers verified that these mechanisms are valid for numerous structures like VPCs with bearded interface and zigzag interface [13, 14].

Now that the efficient transmission of the waveguide against the sharp corner can be explained, one significant issue is the relationship between the robustness of the PhC cavity and the projected band eigen mode profile in k space, or the location of the CPS. VPC cavity shows superiorities including chiral excitation, the ability to generate comb-like resonant modes, and robustness against complex cavity shapes. By exciting the VPC cavity with the chiral source, the spin-momentum locking effect can be realized which can be applied in quantum area [15]. VPC cavity can generate a series of resonant modes, and between each two nearest modes the Fabry-Perot (FP) mode and the corner-localized mode are strongly suppressed [16–19], which are different from the modes in traditional PhC ring resonator [20]. Benefiting from the robust transmission of the waveguide, comb-like modes can be maintained when the cavity shape varies [21, 22]. However, the physical mechanisms behind these phenomena are unclear yet, and whether these traits can be realized without VPC requires further investigation.

In this work, we proposed a PhC cavity based on a line-defect PhC waveguide supercell. We find that the proposed cavity can generate robust comb-like resonant modes. The CPS theory demonstrated in Ref. [9, 13] is adopted here to illustrate the principle of robustness. This work is expected to deepen the understanding of the

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topological photonic device for researchers.

II. THEORY AND MODEL

First, we consider the structure in Figs. 1(a) and 1(b). The PhCs are constituted by the square holes arranged in 45° tilt square lattice. The holes are etched in a suspended silicon membrane. The PhC waveguides are constructed by introducing a line defect into an unperturbed PhC or removing one row of holes in a bulk PhC. The sharp bend can be placed at the even column or the odd column, and consequently, there are two configurations. The red dashed box indicates the supercell for obtaining the projected band. The cyan dashed lines indicate the mirror symmetry axis. a_0 is defined as the constant, and a_1 is defined as the distance between the two nearest holes vertically or horizontally as shown in the inset of Fig. 1(a). The width of the supercell is also equal to a_1 . We set N to 9 so the length of the supercell is $9a_1$. d is the side length of the square hole. Here we set $a_0 = 500$ nm, $a_1 = 500 \times 1.4142$ nm = 707.1 nm, and $d = 360$ nm. The refractive index of the silicon is set to 3.47 and the thickness of the silicon membrane is set to 250 nm. The ratio of the air to the dielectric in a unit cell is 51.84 % to 48.16 %. The ratio is feasible in the practical experiment because PhC with similar air to the dielectric ratio of 50 % to 50 % in PhC membrane has been already successfully fabricated to investigate the corner state [23]. Moreover, the structure in previous work also utilized the square air holes arranged in the square lattice [23]. Consequently, the structure in Fig. 1(a) is compatible with the current fabrication technique. In Fig. 1 the blue arrows indicate the directions of the input waves and output waves. The PhCs in this work are simulated using the three-dimensional finite difference time domain (3D-FDTD) method with perfect-matched-layer (PML) as the boundary condition. Only quasi-transverse electric (quasi-TE) modes are considered owing to that for hole-type PhC it is easier to generate band gap for quasi-TE modes. The structure here is modified from the one in Ref.[9]. However, it should be noted that 3D-FDTD instead of 2D-FDTD simulation is considered here since the membrane structure is selected, which implies that the cavity here is compatible with the current Silicon-on-Insulator (SOI) process and can be integrated on chip.

The transmission spectra of the structures in Figs. 1(a) and 1(b) are simulated. The results are shown in Figs. 1(c) and 1(d) while the transmission spectra of the straight waveguide are also simulated for comparison. The black (red) line denotes the condition of the straight (bend) waveguide. The yellow rectangle indicates the wavelength region with the single mode. It can be inferred that the energy can smoothly propagate through 90° sharp corners over a large bandwidth when the symmetry axis crosses the holes. As for the structure with the symmetry axis crossing between the holes, the energy cannot propagate through sharp corners. We

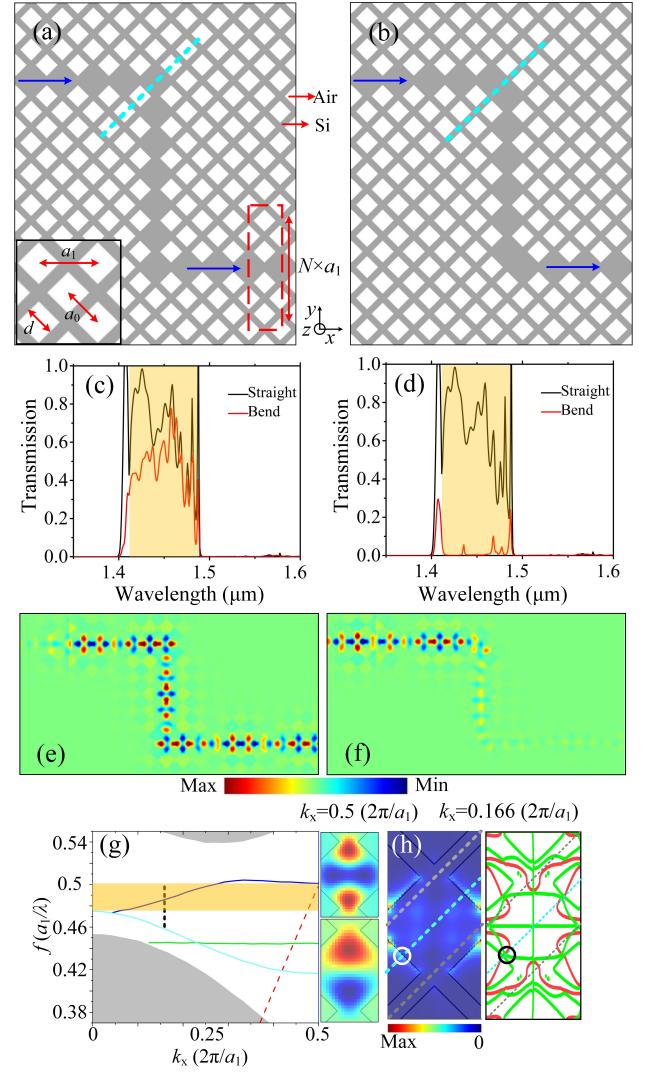


FIG. 1. PhC waveguide with the symmetry axis (a) crossing the holes and (b) crossing between the holes. Inset: The enlarged view of holes. Transmission spectrum of the PhC waveguide with the symmetry axis (c) crossing the holes and (d) crossing between the holes. H_z profiles of the PhC waveguide with the symmetry axis (e) crossing the holes and (f) crossing between the holes at 1461 nm wavelengths. (g) Projected band diagram of the proposed PhC waveguide, where the blue band possesses the superiority of the robust transmission. The yellow rectangle indicates the single-mode frequency region, the dashed red line denotes the light line, and the dashed black line indicates the position of $k_x = 0.16 (2\pi/a_1)$. Inset: H_z profiles of the modes in the blue band (top) and the cyan band (bottom) at $k_x = 0.5 (2\pi/a_1)$. (h) Electric intensity $|E|$ profile (left) and zero-value isolines of Stokes parameters (right) of the mode in the blue band at $k_x = 0.166 (2\pi/a_1)$, where the cyan/grey dashed line denote the symmetry axes, and the circle marks the position of the CPS. The red line dots the zero-value isolines for S_1 parameter, and the green line dots the zero-value isolines for S_2 parameter.

demonstrated the H_z profiles of both PhC waveguides at

1461 nm, and the results are exhibited in Figs. 1(e) and 1(f). The results indicate that, for the structure in Figs. 1(a) most energy can pass through the sharp corners and the in-plane loss of energy is negligible. However, for the structure in Fig. 1(b) most energy is reflected to the input port and only negligible energy can propagate to the output port.

The projected band diagram based on the supercell in Fig. 1(a) is shown in Fig. 1(g). The gray region includes the bulk modes. The cyan band and the blue band intersect at one point. The green band shows almost no variation as k_x increases. The blue band possesses the ability to pass through sharp corner once the symmetry axis crosses the holes. The frequency range marked by the yellow rectangle corresponds to the wavelength ranges in Figs. 1(c) and 1(d). Here we are only interested in the single-mode region. The dashed red line denotes the light line, and the dashed black line indicates the position of $k_x=0.166$ ($2\pi/a_1$). The H_z profiles of the modes in the blue band and the cyan band at $k_x=0.5$ ($2\pi/a_1$) are shown in the inset of Fig. 1(g). The field in the top figure shows even symmetry along y axis while the field in the bottom figure shows odd symmetry along y axis. The contour of the holes is also shown. According to previous work [24], the field in the bottom figure is not easy to be excited by plane-wave source. Consequently, the transmission efficiency in the wavelength range of the cyan band is negligible.

In order to explain the transmission efficiency difference between Figs. 1(c) and 1(d), the CPS theory proposed in Ref.[9, 13] are adopted. The Stokes parameters S_0 , S_1 and S_2 of TE modes are defined as [9, 13, 25] :

$$\begin{aligned} S_0(r) &= |E_x(r)|^2 + |E_y(r)|^2 \\ S_1(r) &= \frac{|E_x(r)|^2 - |E_y(r)|^2}{S_0(r)} \\ S_2(r) &= \frac{2\text{Re}[E_x^*(r)E_y(r)]}{S_0(r)} \end{aligned} \quad (1)$$

In this work, we use five monitors at different heights in z direction to monitor the Stokes parameters. The total Stokes parameter is averaged. Fig. 1(h) shows the $|E|$ profile and the Stokes parameter zero-value isolines at $k_x=0.166$ ($2\pi/a_1$) in blue band. The red line dotes the zero-value isolines for S_1 parameter, and the green line dotes the zero-value isolines for S_2 parameter. We choose $k_x=0.166$ ($2\pi/a_1$) because this point is located at the middle frequency of the blue band, while the higher frequency band is the multi-mode region and the lower frequency band is the slow-light region. We believe this point is representative. Usually, the intersections of the green line and red line indicate the CPSs [9]. If the CPS lies on the mirror-symmetry axis of the bend, the CPS in the forward propagating wave and the downward propagating wave share the same distribution. Otherwise, the CPS' distribution is reverted in the forward and downward propagating waves, thus not matching each other [9, 13]. The detailed explanation can be

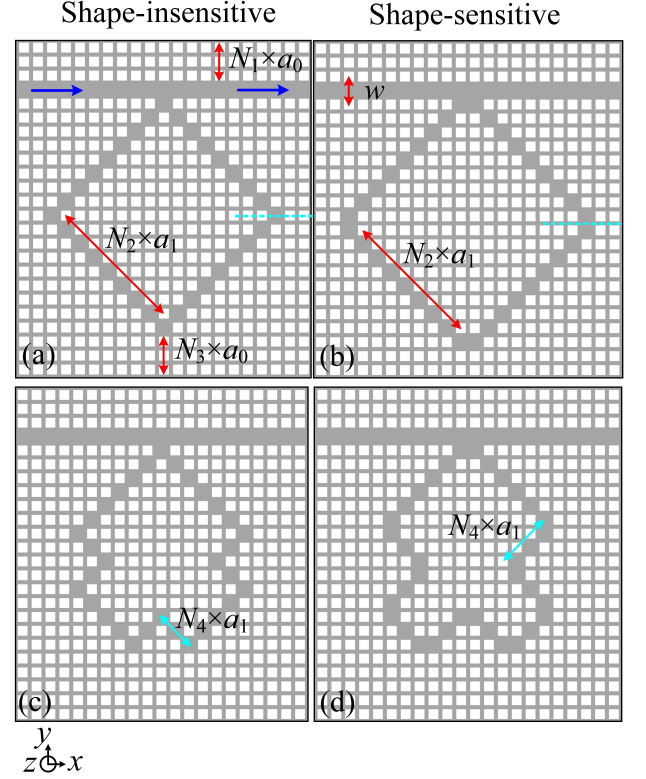


FIG. 2. Schematic of two PhC ring resonators with bus waveguides. The symmetry axes are (a) crossing the holes and (b) crossing between the holes. The former is shape-insensitive while the latter is shape-sensitive. Schematic of disturbed (c) shape-insensitive PhC ring resonators and (d) shape-sensitive PhC ring resonators. It should be noted that the structures in these figures are only for better visualization, and larger structures are used in our real simulations.

referred to Ref.[9, 13]. For the structure in Fig. 1(a), the symmetry axis crosses two CPSs, while the $|E|$ field intensity is not negligible in CPSs. For convenience, we use the white/black circle to mark the position of one CPS in $|E|$ profile/Stokes parameters zero-value isolines distribution). For the structure in Fig. 1(b), the symmetry axis crosses some CPSs, but the field intensities of these CPSs are negligible. The above results indicate that the structure in Fig. 1(a) is suitable for establishing a robust PhC cavity.

III. RESULTS AND DISCUSSION

In order to illustrate the physical mechanism of the robust PhC cavity, a cavity must be simulated using the structure in Fig. 1(a), and its resonant properties must be analyzed. For the silicon VPC, the common excitation approach is the on-chip injection [26, 27] while for GaAs VPC the space-light excitation is commonly adopted [17].

The PhC ring cavity is constructed by the line defect as shown in Figs. 2(a) and 2(b). A bus waveguide is

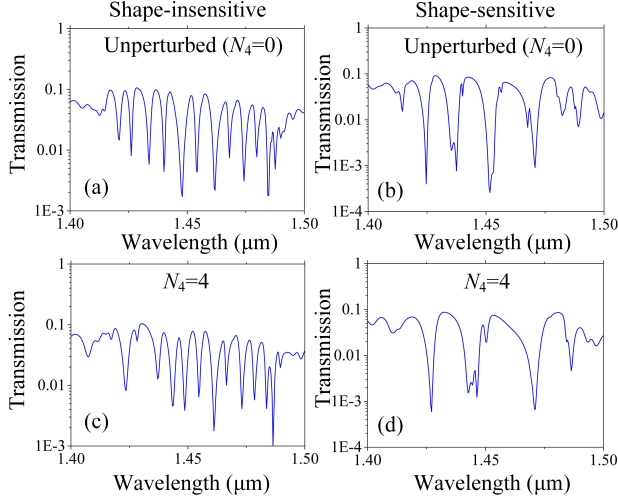


FIG. 3. Transmission spectra of the unperturbed (a) shape-insensitive PhC ring resonators and (b) shape-sensitive PhC ring resonators. Transmission spectra of the perturbed (c) shape-insensitive PhC ring resonators and (d) shape-sensitive PhC ring resonators.

introduced. The energy is excited from the left port and collected from the right port. First, we remove one row of holes to form the bus waveguide. We find that the multi-mode region occupies most regions of the band gap. Consequently, the width of the bus waveguide is decreased. $w = 1.5a_0$ is defined as the distance between two holes near the bus waveguide in the vertical direction. The PhCs surround the bus waveguide and the cavity are set as reflection mirrors. The length of top reflection mirrors is set to $N_1 \times a_0$, and we set $N_1 = 5$. The side length of the cavity is $N_2 \times a_0$. The cavity constructed with lossless corners is defined as shape-insensitive cavity while the cavity constructed with lossy corners is defined as shape-sensitive cavity. For the shape-insensitive cavity, we set $N_2 = 12$. For the shape-sensitive cavity, we set $N_2 = 13$ owing to different configurations at the corners. The length of other reflection mirrors is set to $N_3 \times a_0$, and we set $N_3 = 3$. In order to verify the robustness of the cavity in Figs. 2(a), we modify the shape of the proposed cavity and introduce the complex boundaries as shown in Fig. 2(c), the modified cavity based on Fig. 2(b) is also shown in Fig. 2(d) for comparison. The cavities with complex boundaries are distorted by changing the orientations of three corners. After modification more cavity corners are introduced since for one side of the cavity, only the holes with the side length of $N_4 \times a_1 = 4 \times a_1$ vary their trend.

The transmission spectra of the cavities in Fig. 2 are shown in Fig. 3. The superiority of the shape-insensitive cavity can be directly discovered. A series of comb-like resonant modes are located between the slow-light region and the multi-mode region. The modes are almost the same as those in Ref. [26] which utilized VPCs. Between each two nearest resonant modes, the FP mode and

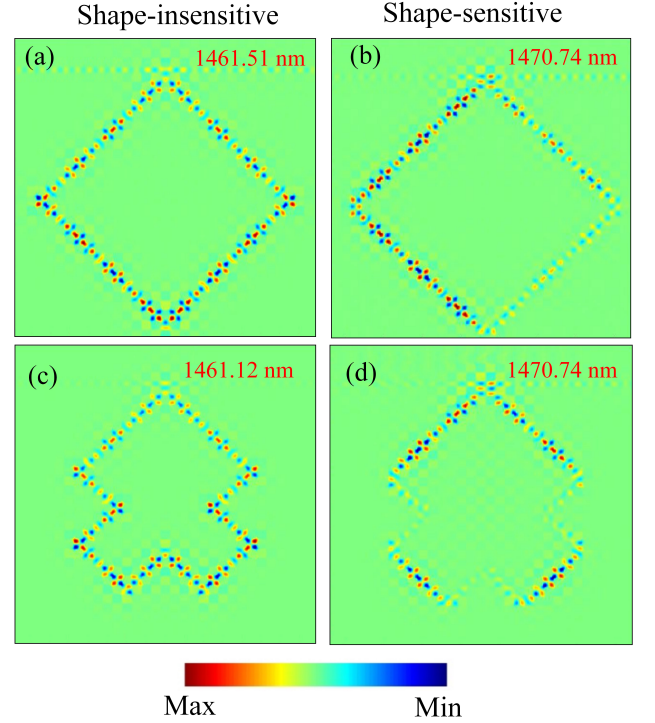


FIG. 4. H_z profiles of modes in unperturbed (a) shape-insensitive cavity at 1461.51 nm and (b) shape-sensitive cavity at 1470.74 nm. H_z profiles of modes in perturbed (c) shape-insensitive cavity at 1461.12 nm and (d) shape-sensitive cavity at 1470.74 nm.

corner-localized mode are suppressed unlike traditional PhC ring resonator [20]. As for the shape-sensitive cavity, the disturbed modes are located everywhere. This phenomenon is also observed in Ref. [15]. Moreover, as the complex cavity boundaries are introduced, comb-like resonant modes can be maintained for the shape-insensitive cavity, while for the shape-sensitive cavity, the transmission spectrum indicates astonishing variation. In Fig. 2(d) we can observe that many new disturbed modes are introduced.

In order to clearly understand the robustness of the cavities in Fig. 2(a), the H_z profiles of the mode of 1461.51 nm of the shape-insensitive cavity, and the H_z profiles of the mode of 1470.74 nm of the shape-sensitive cavity are shown in Figs. 4(a)-4(b). The fields are obtained in $16 \mu\text{m} \times 16 \mu\text{m}$ area. It can be noticed that for the shape-insensitive cavity the energy can propagate along the whole cavity edges, while for the shape-sensitive cavity, the energy reflected through three cavity edges. As the complex boundaries are introduced, the H_z profiles of the mode of 1461.12 nm of the shape-insensitive cavity, and the H_z profiles of the mode of 1470.74 nm of the shape-sensitive cavity are simulated and the results are shown in Figs. 4(c)-4(d). The fields indicate that the energy can propagate through the added corners with negligible loss for the shape-insensitive cav-

ity, while strong reflection can be observed between corners for the shape-sensitive cavity.

It should be noted that the bichromatic PhC [28, 29] and PhC microring cavity [30, 31] can also generate comb-like resonant modes with the disturbed modes suppressed. However, bichromatic PhC and PhC microring cavity do not exhibit robustness against cavity shape. The superiority of robustness is beneficial for the on-chip integration. Owing to that the shape of the cavity can be transformed into lots of different shapes which does not influence its basic properties, the free space of the photonic chip can be efficiently utilized and more photonic devices can be integrated. One drawback of the proposed structure is that its modal volume still requires improvement compared with bichromatic PhC and PhC microring cavities. Another drawback of the proposed cavity is that it works inside the light line, which leads to out-of-plane loss. The loss will degrade the quality factor of the cavity. However, now that the robustness of the PhC ring resonator is related to its Stokes parameters distribution instead of the inversion symmetry or the VCNs, we believe such phenomena can be found in more colorful structures.

The robustness of the shape-insensitive cavity actually derives from the ability to pass the sharp corner of the PhC waveguide. In the former part, we explain that the Stokes parameters distributions determine the locations of the CPSs. As the symmetry axis together with the intense electric field cross the CPSs, the waves in straight waveguides near the corner are matched. Consequently, the robust transmission can be realized. For the shape-insensitive cavity, the waveguide mode propagates in the

cavity edge with negligible in-plane loss which results in the suppression of disturbed modes. For the shape-sensitive cavity, the waveguide mode suffers from strong reflection at the corner, the reflected wave has more possibility to form FP resonance or corner-localized resonance. Meanwhile, once the complex cavity shape is introduced, more lossy corners will be introduced, and the transmission spectra will be distorted.

IV. CONCLUSION

In conclusion, we proposed a PhC ring resonator that can generate robust comb-like modes, and the physical mechanism behind the robustness is analyzed. The robustness derives from the overlapping among the CPSs, the symmetry axis of the PhC cavity, and the intense electric field. The proposed cavity does not possess the ability of valley-momentum locking, thus unidirectional transmission will not occur once excited by orthogonal dipoles with the different phases. The side length of the cavity can be flexibly adjusted while the desired free spectral range can be realized. The suppression of perturbed modes of the cavity is an important merit in sensing, lasing and nonlinear areas. In the future, large-bandwidth, single-mode, and robust waveguide band with no out-of-plane loss can be expected. Whether the CPS theory can be extended to the PhC with photonic quantum spin-Hall effect and the phononic crystal will also be our future works.

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